METHOD OF ANALYSIS OF STOCHASTIC NONSTATIONARY HEAT CONDUCTION EQUATIONS

A. G. Madera

UDC 536.2(075)

A method is proposed for determination of nonstationary fields of mathematical expectation, second moments, and dispersion of a stochastic nonstationary temperature field for inhomogeneous bodies of an arbitrary form and dimension with arbitrary stochastic boundary conditions.

The stochastic distribution of a nonstationary temperature in a body takes place when its controlling factors, namely, internal heat sources, surface heat sources, ambient temperature or initial temperature in the body, coefficients of heat transfer from the body surface to a medium, are stochastic. The stochastic character of these factors, nonstationary in the general case, is conditioned by random fluctuations of external effects and the surrounding medium. The analysis of random fluctuations of the power consumed by different technical systems from external sources of power supply, the medium temperature, the heat fluxes incident on an object surface from outside, e.g., solar radiation, and so on shows that they are manifested, as a rule, in the form of random deviations about some mean value and their nearest, with respect to time, amplitudes are, in fact, not correlated between each other. This allows one to model them as stochastic white noises or as Wiener stochastic processes with independent increments. The stochastic nonstationary temperature fields are described by partial differential equations and boundary conditions in which all the functions and coefficients are random functions of time and coordinates.

In engineering applications such statistical measures as the fields of mathematical expectations and second moments of stochastic temperature distributions describe sufficiently completely the stochastic processes of heat transfer in different objects. The fields of those statistical measures allow determination of the intervals within which real temperatures change.

The current method of the stochastic Green function [1, 2] makes it possible to model stochastic temperature fields in the cases where the Green function may be found analytically, which restricts its application.

In the present work, the numerical method is proposed for determination of the nonstationary fields of statistical measures, i.e., the mathematical expectation and second moments of the stochastic nonstationary temperature field in bodies of complex geometry and any dimension described by the stochastic nonstationary partial differential heat conduction equation. The method rests on the theory of stochastic differential Ito equations as applied to the system of stochastic ordinary differential equations obtained after approximation of the initial stochastic mathematical model by its discrete analog.

Stochastic Mathematical Model and Its Discrete Analog. The stochastic nonstationary temperature field $u(x, t, \omega)$ in the three-dimensional inhomogeneous body $D \in R^3$, $x = (x_1, x_2, x_3)$ with the boundary S is described by the following equation:

$$\rho c \frac{\partial u}{\partial t} - A(x, t, \omega) u = f(x, t, \omega), \ (x, t, \omega) \in D \times [0, T] \times \Omega, \tag{1}$$

with the boundary condition

$$B(x, t, \omega) u = q(x, t, \omega), \ (x, t, \omega) \in S \times [0, T] \times \Omega, \tag{2}$$

and the initial one

$$u(x, 0, \omega) = u_0(x, \omega), \ (x, \omega) \in D \times \Omega, \tag{3}$$

Institute for Cybernetics Problems, Russian Academy of Sciences, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 64, No. 2, pp. 244-251, February, 1993. Original article submitted April 22, 1992.

1062-0125/93/6402-0195$12.50

where f(x, t, ω), q(x, t, ω) are random functions of time t and coordinates x modeling the stochastic internal and surface heat sources; A(x, t, ω) is the stochastic operator of the equation

$$Au = \sum_{i=1}^{3} \frac{\partial}{\partial x_i} \left(\lambda(x, t) \frac{\partial u}{\partial x_i} \right) - b(x, t, \omega) u; \tag{4}$$

B(x, t, ω) is the stochastic operator of boundary conditions of one of the three kinds:

$$Bu = u, \; Bu = \lambda(x, t) \frac{\partial u}{\partial n}, \quad Bu = \lambda(x, t) \frac{\partial u}{\partial n} + \alpha(x, t, \omega) u; \tag{5}$$

λ(x, t), ρ, c are the thermal conductivity, density, and specific heat, respectively; b(x, t, ω), α(x, t, ω) are random functions of t and x; α(x, t, ω) models the stochastic coefficient of heat transfer from the body surface to the medium; $u_0(x, \omega)$ is the random field of the initial temperature. For the sake of clarity, the ambient temperature in the third-kind boundary conditions is assumed equal to zero. Account of a nonzero temperature, which may be stochastic, presents no difficulties.

The random functions f(x, t, ω), q(x, t, ω), b(x, t, ω), α(x, t, ω) represent independent Gaussian white noises with the continuous-in-time mathematical expectations f(x, t), q(x, t), b(x, t), α(x, t) and the continuous-in-time and limited intensities $D_f(x, t)$, $D_q(x, t)$, $D_b(x, t)$, $D_\alpha(x, t)$, respectively. We write the random functions f, q, b, α as sums of their mathematical expectations and stochastic centered fluctuations with zeroth mathematical expectations, i.e.,

$$f = \bar{f} + f^0, \; q = \bar{q} + q^0, \; b = \bar{b} + b^0, \; \alpha = \bar{\alpha} + \alpha^0. \tag{6}$$

The stochastic fluctuations $f^0 = f^0(x, t, \omega)$, $q^0 = q^0(x, t, \omega)$, $b^0 = b^0(x, t, \omega)$, $\alpha^0 = \alpha^0(x, t, \omega)$ are Gaussian white noises in time t with zeroth mathematical expectations and are described as the formal derivative with respect to time from Wiener processes with independent increments. The stochastic fluctuations f^0, q^0, b^0, α^0 will be considered to be statistically independent for any two points $x = (x, x_2, x_3) \in \bar{D}$ ($\bar{D} = D + S$). The Wiener processes $dW_f = f^0dt$, $dW_q = q^0dt$, $dW_b = b^0dt$, and $dW_\alpha = \alpha^0dt$ are characterized by the following relations:

$$M\{dW_f\} = 0, \; M\{dW_q\} = 0, \; M\{dW_b\} = 0, \; M\{dW_\alpha\} = 0$$

and for any two points x, y ∈ $\bar{D}$

$$\begin{aligned} M\{dW_f(x, t, \omega)\, dW_f(y, t, \omega)\} &= \delta_{xy} D_f(x, t)\, dt, \; x, y \in D, \\ M\{dW_q(x, t, \omega)\, dW_q(y, t, \omega)\} &= \delta_{xy} D_q(x, t)\, dt, \; x, y \in S, \\ M\{dW_b(x, t, \omega)\, dW_b(y, t, \omega)\} &= \delta_{xy} D_b(x, t)\, dt, \; x, y \in D, \\ M\{dW_\alpha(x, t, \omega)\, dW_\alpha(y, t, \omega)\} &= \delta_{xy} D_\alpha(x, t)\, dt, \; x, y \in S; \end{aligned}$$

where $\delta_{xy} = 1$ if x = y, $\delta_{xy} = 0$ if x ≠ y; M{·} is the operator of mathematical expectation.

For the sake of concreteness and generality of presentation we shall consider the third-kind boundary condition.

Substituting (6) into the initial mathematical model (1)-(3), we obtain

$$\rho c \frac{\partial u}{\partial t} - \bar{A}(x, t) u + b^0(x, t, \omega) u = \bar{f}(x, t) + f^0(x, t, \omega), \tag{7}$$

$$(x, t, \omega) \in D \times [0, T] \times \Omega,$$

$$\bar{B}(x, t) u + \alpha^0(x, t, \omega) u = \bar{q}(x, t) + q^0(x, t, \omega), \tag{8}$$

$$(x, t, \omega) \in S \times [0, T] \times \Omega,$$

$$u(x, 0, \omega) = u_0(x, \omega), \; (x, \omega) \in D \times \Omega, \tag{9}$$

where Ā(x, t) and B(x, t) are the mathematical expectations of operators of the equation and the third-kind boundary conditions, equal, respectively, to

$$\bar{A}(x, t) = \sum_{i=1}^{3} \frac{\partial}{\partial x_i} \left(\lambda(x, t) \frac{\partial}{\partial x_i} \right) - \bar{b}(x, t), \tag{10}$$

$$\bar{B}(x, t) = \lambda(x, t) \frac{\partial}{\partial n} + \bar{\alpha}(x, t). \tag{11}$$

We pass to obtaining the discrete analog of stochastic mathematical model (7)-(11). For this, we employ the integrointerpolation method [4] or the control volume method [5], which make it possible to obtain conservative homogeneous difference schemes with second order of accuracy in the class of discontinuity coefficients.

We subdivide the region $D \in R^3$ into n control volumes by equidistant planes parallel to the axes x_1, x_2, x_3. Let one node be centrally located inside each control volume. We designate the set of all internal nodes of the region as D_h and its appropriate internal control volumes as D_i, $i = 1, 2, \ldots, n_1$. The boundary control volumes are designated as S_i, $i = 1, 2, \ldots, n_2$; each control volume S_i has one node being only on the surface S of the region D; the set of all n_2 boundary nodes is designated by S_h. Also, we introduce the set of near-boundary nodes D_{*h}, formed by the internal nodes from the set D_h nearest to the boundary S but not belonging to it [4]. The total number of the nodes $n = n_1 + n_2$ from the set $D_h + S_h$ will be called a network.

Following the control volume method, we integrate Eq. (7) for each $\omega \in \Omega$ with respect to all n_1 internal control volumes D_i. As a result, we arrive at

$$\rho c \frac{du_{h1}}{dt} - \bar{A}_h(t) u_h + b^0_{h1}(t, \omega) u_{h1} = \bar{f}_{h1}(t) + f^0_{h1}(t, \omega), \tag{12}$$

where $u_h = u_h(t, \omega)$ is a random function of the network, determined at the network nodes $D_h + S_h$ representing the n-dimensional random column-vector $u_h = (u_1(t, \omega) \ldots u_n(t, \omega)$; $u_{h1} = u_{h1}(t, \omega)$ is a random function of the network determined at the internal network nodes D_h, representing an n_1-dimensional random vector-column; $b^0_{h1}(t, \omega)$, $f^0_{h1}(t, \omega)$ are random functions of the network determined at the internal nodes D_h, representing the n_1-dimensional random column-vectors $b^0_{h1}(t, \omega) = (b^0_1(t, \omega) \ldots b^0_{n1}(t, \omega)^T$, $f^0_{h1}(t, \omega) = (f^{0n1}(t, \omega) \ldots f^0_{n1}(t, \omega)^T$; here

$$b^0_i(t, \omega) = \frac{1}{V_{D_i}} \int_{D_i} b^0_i(x, t, \omega)\, dD_i, \quad f^0_i(t, \omega) = \frac{1}{V_{D_i}} \int_{D_i} f^0_i(x, t, \omega)\, dD_i; \tag{13}$$

V_{Di} is the volume of D_i; $b_{h1}(t) = (b_1 t) \ldots b_{n1}(t))^T$, $f_{h1}(t) = (f_1(t) \ldots f_{n1}))^T$ are determinate n_1-dimensional column-vectors; here

$$\bar{b}_i(t) = \frac{1}{V_{D_i}} \int_{D_i} \bar{b}_i(x, t)\, dD_i, \quad \bar{f}_i(t) = \frac{1}{V_{D_i}} \int_{D_i} \bar{f}_i(x, t)\, dD_i; \tag{14}$$

$\bar{A}_h(t)$ is a determinate nonstationary difference operator leaving the network function u_h only at the neighboring nearest nodes surrounding each internal node from D_h; T stands for transposition.

System of equations (12) consists of n_1 equations for n unknown random temperatures $u_i(t, \omega)$, $i = 1, 2, \ldots, n$, at all the nodes from $D_h + S_h$. To obtain the lacking n_2 equations, we perform integration of (7) over all the boundary control volumes S_i, $i = 1, 2, \ldots, n_2$, with account of boundary conditions (8). As a result, we have

$$\rho c \frac{du_{h2}}{dt} - \bar{B}_{h*}(t) u_{h*} + \bar{b}_{h2}(t) u_{h2} + \bar{\alpha}_{h2}(t) u_{h2} + b^0_{h2}(t, \omega) u_{h2} + \tag{15}$$
$$+ \alpha^0_{h2}(t, \omega) u_{h2} = \bar{f}_{h2}(t) + \bar{q}_{h2}(t) + f^0_{h2}(t, \omega) + q^0_{h2}(t, \omega),$$

where $u_{h2} = u_{h2}(t, \omega)$ is a random function of the network determined at the boundary nodes from S_h, representing an n_2-dimensional random column-vector; $b_{h2}(t)$, $f_{h2}(t)$, $\alpha_{h2}(t)$, $q_{h2}(t)$ are determinate n_2-column-vectors, where $b_i(t)$ and $f_i(t)$ are determined by formulas like (14), $\alpha_{h2}(t) = \alpha 121(t) \ldots \alpha_{n2}(t))^T$, $q_{h2}(t) = (q_1(t) \ldots q_{n2}(t))^T$ are determinate network functions determined at the boundary nodes from S_h; B_{h*} is determinate nonstationary difference operator leaving the network function u_h only at the boundary nodes from D_h and near-boundary nodes from D_{h*}, surrounding each boundary node from S_h; $u_{h*} = u_{h*}(t, \omega)$ is a network function determined at the boundary and near-boundary nodes from $D_h + D_{h*}$; $b^0_{h2}(t, \omega)$, $f_{0h2}(t, \omega)$, $\alpha^0_{h2}(t, \omega)$, $q^0_{h2}(t, \omega)$, $q^0_{h2}(t, \omega)$ are random n_2-dimensional columns-vectors, where $b^0_i(t, \omega)$ and $f^0_i(t, \omega)$ are determined by expressions of the form (13), and $\alpha^0_{h2}(t, \omega) = (\alpha^0_1(t, \omega) \ldots \alpha^0_{n2}(t, \omega))^T$, $q^0_{h2}(t, \omega) = (q^0_1(t, \omega) \ldots q^0_{n2}(t, \omega))^T$ are random functions of the network determined at the boundary nodes S_h.

We introduce a network function of Gaussian white noises N(t) with the dimension $m = 2(n_1 + n_2)$ determined on the set $D_h + S_h$, $N(t) = (N_1(t) \ldots N_m(t))^T = (f^0_1 \ldots f^0_{n1}, b^0_1 \ldots b^0_{n1}, \alpha^0_1 \ldots \alpha^0_{n2}, q^0_1 \ldots q^0_{n2})^T$, the network function of Wiener processes determined on the set $D_h + S_h$, $dW(t) = (dW_1(t) \ldots d^{**}{}_n(t))^T = (dW_{f1} \ldots dW_{fn1}, dW_{b1} \ldots dW_{bn1}, dW_{\alpha 1} \ldots dW_{\alpha n1}, dW_{q1} \ldots dW_{qn2})^T$ and the diagonal $m \times m$-matrix D(t), on whose diagonal we have the network functions $D_{f1}(t), \dots, D_{fn1}(t), D_{b1}(t), \dots, D_{bn1}(t), D_{\alpha 1}(t), \dots, D_{\alpha n2}(t), D_{q1}(t), \dots, D_{qn2}(t)$. For the stochastic vector $dW(t)$ the following relations are valid:

$$dW(t) = N(t)\,dt,$$
$$M\{dW(t)\} = 0,$$
$$M\{dW(t)\,dW^{T}(t)\} = D(t)\,dt.$$

We obtain the discrete analog of stochastic mathematical model (1)-(3) by combining Eqs. (12), (15) into one stochastic matrix differential-difference equation

$$\rho c\frac{du_h}{dt} = \bar{\mathcal{A}}_h(t)\,u_h + \bar{\varphi}_h(t) + \mathcal{B}_h(t,\ u_h)\,N(t), \tag{16}$$

$$u_h(0,\ \omega) = u_{0h}(\omega), \tag{17}$$

where $\bar{A}(t)$ is a determinate nonstationary $n \times n$-matrix of the known coefficients; $\mathcal{B}_h(t, u_h)$ is a stochastic nonstationary $n \times m$-matrix dependent on the network random function $u_h = u_h(t, \omega)$; $\varphi_h(t)$ is a determinate known n-dimensional column-vector including the vectors $f_{h1}(t)$, $f_{h2}(t)$, $q_{h2}(t)$; $u_{0h}(\omega)$ is a network random function of initial temperatures.

Determination of Statistical Measures. Integrating formally Eq. (16) with respect to time within the limits from 0 to t with the initial condition (17), we obtain the stochastic vector-matrix integral Ito equation [3, 6]:

$$u_h(t) = u_{0h} + \int_0^t a(\tau,\ u_h(\tau))\,d\tau + \int_0^t \beta(\tau,\ u_h(\tau))\,dW(\tau), \tag{18}$$

where $a(t, u_h)$ is the stochastic n-dimensional vector function

$$a(t,\ u_h) = \frac{1}{\rho c}\bar{\mathcal{A}}_h(t)\,u_h(t) + \frac{1}{\rho c}\bar{\varphi}_h(t); \tag{19}$$

$\beta(t, u_h)$ is the stochastic $n \times m$-matrix function

$$\beta(t,\ u_h) = \frac{1}{\rho c}\mathcal{B}_h(t,\ u_h). \tag{20}$$

The variable ω here and henceforth is omitted for the sake of brevity.

Stochastic integral equation (18) is equivalent to the vector-matrix stochastic differential Ito equation [6]:

$$du_h(t) = a(t,\ u_h) + \beta(t,\ u_h)\,dW(t),\quad u_h(0) = u_{0h}. \tag{21}$$

As for the functions $a(t, u_h)$ and $\beta(t, u_h)$, we assume that they satisfy the conditions of the theorem of existence and uniqueness [6, 7] of the solution of the stochastic differential Ito equation (21) and are nonpredicting [8].

Reducing initial stochastic mathematical model (1)-(3) to stochastic equations (18) and (21) permits us to apply the Ito theory to them.

We now determine the statistical measures of the stochastic vector $u_h(t, \omega)$, i.e., the n-vector of mathematical expectations $\bar{u}_h(t) = M\{u_h(t, \omega)\}$ and the $n \times n$-matrix of the second initial moments $C(t) = M\{u_h u^T_h\}$. The vector of dispersions $D_u(t)$ of stochastic temperatures $u_h(t, \omega)$ will be equal to the diagonal elements of the matrix of the second central moments $K(t) = C(t) - \bar{u}_h(t)\bar{u}^T_h(t)$.

We introduce the continuous scalar function $\psi(u_h)$ having continuous first derivative with respect to time and first and second derivatives with respect to all elements u_i of the vector u_h, $i = 1, 2, \dots, n$. We apply the Ito formula [3, 8, 9] to the function $\psi(u_h)$, in which u_h satisfies the stochastic equations (18) or (21), and we obtain

$$\frac{d\psi}{dt} = \frac{\partial\psi^T}{\partial u_h}a(t,\ u_h) + \frac{1}{2}\mathrm{tr}\left\{\frac{\partial^2\psi}{\partial u_h \partial u_h}\beta D\beta^T\right\} + \frac{\partial\psi^T}{\partial u_h}\beta N(t), \tag{22}$$

where $\partial\psi^T/\partial u_h = (\partial\psi/\partial u_1 \dots \partial\psi/\partial u_n)^T$ is the n-dimensional vector of the first derivatives of the function ψ over all u_i, u_j of the vector u_h; $\partial^2\psi/\partial u_h\partial u_h = \{\partial^2\psi/\partial u_j\partial u_i\}$ is the $n \times n$-matrix of the second derivatives of the function ψ over all u_i, u_j; $\mathrm{tr}\{\cdot\}$ is the sign of the matrix $\{\cdot\}$ [3].

We assume $\psi(u_h) = u_h(t)$, substitute it into Eq. (22) and apply the operator of mathematical expectation. As a result, we obtain a matrix ordinary differential equation for the vector of mathematical expectation $\bar{u}_h(t)$:

$$\rho c \frac{d\bar{u}_h(t)}{dt} = \rho c a(t, u_h) = \bar{\mathcal{A}}_h(t)\bar{u}_h(t) + \bar{\varphi}_h(t), \qquad \bar{u}_h(0) = \bar{u}_{0h}, \tag{23}$$

where $\bar{u}_{oh}$ is the network function of mathematical expectation of initial temperatures. Let $\psi = u_i(t)u_j(t)$, substitute it into Eq. (22) and apply the operator of mathematical expectation. As a result we obtain the matrix ordinary differential equation for determination of the $n \times n$-matrix of the second initial moments C with the elements $C_{ij}(t) = M\{u_i(t, \omega)u_j(t, \omega)u_j(t, \omega)\}$, $i, j = 1, 2, \dots, n$:

$$\rho c \frac{dC}{dt} = \bar{\mathcal{A}}_h(t)C(t) + C(t)\bar{\mathcal{A}}_h^T(t) + \bar{\varphi}_h(t)\bar{u}_h^T(t) + \bar{u}_h(t)\bar{\varphi}_h^T(t) + \frac{\rho c}{2} M\{\beta D \beta^T\}, \qquad C(0) = C_0, \tag{24}$$

where $C_0 = M\{u_{oh}u^T_{oh}\}$ is the known $n \times n$-matrix of the second initial moments of initial temperatures.

The systems of ordinary differential equations for determination of the statistical measures $\bar{u}_h(t)$ of (23) and C(t) of (24) are determinate and for their solution well developed computer programs may be used.

Now we illustrate application of the above method.

Example. Consider a rod with density ρ, specific heat c and thermal conductivity λ, length ℓ, the area and perimeter of its cross section equal to s and p, respectively, inside which there is a heat source with the intensity $\Phi(x, t)$. At the rod ends zero temperatures are maintained. Heat transfer proceeds between the side surface and the medium with the temperature $t_c(x, t)$ and the stochastic heat transfer coefficient $\alpha(x, t, \omega)$ having the mathematical expectation $\alpha(x, t)$ and the dispersion $D_\alpha(x, t)$.

The mathematical model is of the form

$$\rho c \frac{\partial u}{\partial t} = \lambda \frac{\partial^2 u}{\partial x^2} - b(x, t, \omega)(u - u_c) + \Phi(x, t), \quad \omega \in \Omega,$$

$$u(0) = u(l) = 0,$$

$$u(x, 0) = u_0(x), \quad x \in [0, l],$$

where $b = \alpha p/s$ is the convective coefficient.

We write $\alpha(x, t, \omega)$ in the form $\alpha(x, t, \omega) = \alpha(x, t) + \alpha^0(x, t, \omega)$, where α^0 is the Gaussian white noise, being the formal derivative of the Wiener process $dW_\alpha(x, t) = N(x, t)dt$, $N(x, t) = \alpha^0(x, t, \omega)$; here $M\{dW_\alpha\} = 0$, $M\{dW_\alpha(x, t)dW_\alpha(y, t)\} = \delta_{xy}D_\alpha(x, t)dt$, $x, y \in [0, l]$.

After subdividing the rod length $[0, l]$ into control volumes with the length h and applying the integrointerpolation method, we arrive at the stochastic analog

$$\rho c \frac{du_h}{dt} = \bar{\mathcal{A}}_h(t)u_h(t) + \bar{\varphi}_h(t) + \mathcal{B}_h(t, u_h)N(t),$$

where $u_h(t) = (u_1(t) \dots u_h(t))^T$ is a vector of random temperatures at the rod nodes to be determined;

$$\bar{\mathcal{A}}_h(t) = \begin{bmatrix} -2\delta - \bar{b}_1(t) & \delta & 0\dots0 & 0 \\ \delta & -2\delta - \bar{b}_2(t) & \delta\dots0 & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & 0\dots\delta & -2\delta - \bar{b}_n(t) \end{bmatrix}, \quad \delta = \lambda/h;$$

where $b_i(t) = M\{b_i(t, \omega)\}$ is the mathematical expectation of the convective coefficient at the node i;

$$\bar{\varphi}_h(t) = \begin{bmatrix} \Phi_1(t) + \bar{b}_1(t)u_{c1}(t) \\ \Phi_2(t) + \bar{b}_2(t)u_{c2}(t) \\ \dots \\ \Phi_n(t) + \bar{b}_n(t)u_{cn}(t) \end{bmatrix},$$

$$\Phi_i(t) = M \int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} \Phi(x,\ t)\,dx,\ \bar{b}_i(t) = M \int_{x_{i-\frac{1}{2}}}^{x_{i+\frac{1}{2}}} b(x,\ t,\ \omega)\,dx,\ u_{ci} = u_c(x_i);$$

$B_h(t,\ u_h)$ is a square stochastic diagonal $n \times n$-matrix equal to

$$\mathcal{B}_h(t,\ u_h) = \begin{bmatrix} -u_1(t) + u_{c1}(t) & 0 & \dots & 0 \\ 0 & -u_2(t) + u_{c2}(t) & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & -u_n(t) + u_{cn}(t) \end{bmatrix};$$

$N(t) = (b^0_1(t), \dots, b^0_n(t))^T$ is the n-dimensional vector of Gaussian white independent noises.

According to formulas (23), (24) we obtain matrix equations for determination of the statistical measures of the stochastic vector of temperatures $u_h(t)$:

the vector of mathematical expectation $\bar{u}_h(t)$

$$\rho c \frac{d\bar{u}_h}{dt} = \bar{\mathcal{A}}_h(t)\,\bar{u}_h(t) + \bar{\varphi}_h(t),\ \bar{u}_h(0) = \bar{u}_{0h};$$

the correlation matrix C(t)

$$\rho c \frac{dC(t)}{dt} = \bar{\mathcal{A}}_h(t)\,C(t) + C(t)\,\bar{\mathcal{A}}_h(t) + \bar{\varphi}_h(t)\,\bar{u}_h^T(t) + \bar{u}_h(t)\,\bar{\varphi}_h^T(t) +$$

$$+ \frac{1}{2\rho c}\{\mathrm{diag}\, C(t) + U(t)\}\, D_b(t),\ C(0) = C_0,$$

where diag C(t) is a diagonal matrix composed of the diagonal elements of the matrix C(t); U(t) is a diagonal matrix with the elements $u^2_{ci}(t) - 2\bar{u}_i(t)u_{ci}(t)$; $D_b(t)$ is the diagonal matrix of dispersions of the convective coefficients $b_i(t, \omega)$, $i = 1, 2, \dots, n$, the i-th element of the matrix $D_b(t)$ is equal to $D_{bi}(t) = D_{\alpha i}(t)p/s$.

Conclusion. The equations obtained allow one to determine numerically the nonstationary distributions of mathematical expectation and second moments of the stochastic nonstationary temperature field described by the three-dimensional stochastic nonstationary heat conduction equation in partial derivatives with arbitrary boundary conditions. The region for which nonstationary statistical measures are to be determined is inhomogeneous and of arbitrary configuration. The method is easily programmable, and it may serve as a basis for developing program packages for analysis of nonstationary stochastic three-dimensional temperature fields of complex geometry.

NOTATION

$u(x, t, \omega)$, stochastic nonstationary temperature field; Ω, space of elementary events ω; $A(x, t, \omega)$, stochastic nonstationary operator of the heat conduction equation; $B(x, t, \omega)$, stochastic nonstationary operator of boundary conditions; $f(x, t, \omega)$, random function of time t and coordinates x with mathematical expectation $f(x, t)$ and dispersion $D_f(x, t)$; f^0, stochastic fluctuation, being Gaussian white noise; dW_f, Wiener process; $u_h(t, \omega)$, network random function of temperature determined at the network nodes of the region; $\bar{A}_h(t)$, difference nonstationary operator; $\bar{u}_h(t)$, C(t), vector of mathematical expectation and matrix of second initial moments of the stochastic vector of temperatures $u_h(t)$.

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